Danish Fire Data Example
based on
Mikosch: Non-Life Insurance Mathematics (pp. 32-28)

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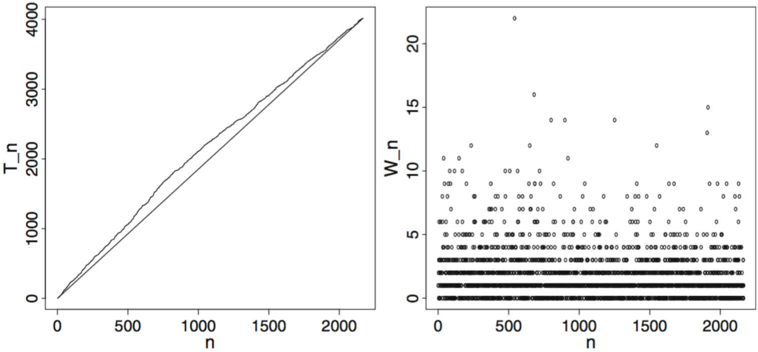
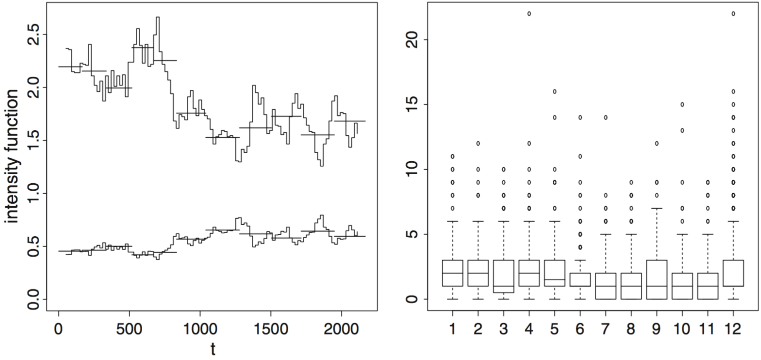
Arrival times and Interarrival times



- Danish fire insurance data 1980 - 1990
- $n = 2167$ observations
- overall sample mean = 1.85

Intensity



- Left upper: Annual expected inter-arrival times (moving averages), lines correspond to expected inter-arrival times
- Left lower: Estimates of Poisson intensities (note that it increases in time)
- Right: Boxplot of interarrival times for different years

Data - Interarrival Times

year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	all
sample size	166	170	181	153	163	207	238	226	210	235	218	2 167
min	0	0	0	0	0	0	0	0	0	0	0	0
1st quartile	1	1	0.75	1	1	1	0	0	0	0	0	1
median	2	2	1	2	1.5	1	1	1	1	1	1	1
mean	2.19	2.15	1.99	2.37	2.25	1.76	1.53	1.62	1.73	1.55	1.68	1.85
$\widehat{\lambda} = 1/\text{mean}$	0.46	0.46	0.50	0.42	0.44	0.57	0.65	0.62	0.58	0.64	0.59	0.54
3rd quartile	3	3	3	3	3	2	2	2	3	2	2	3
max	11	12	10	22	16	14	14	9	12	15	9	22

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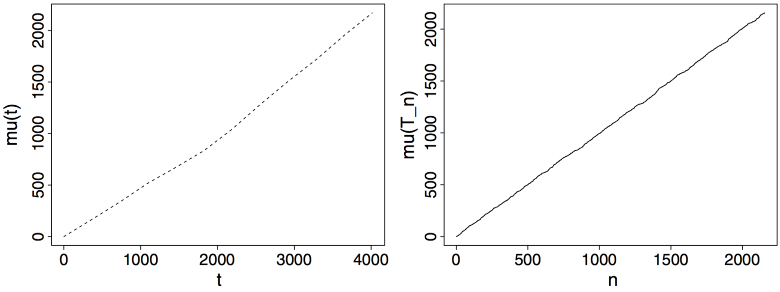
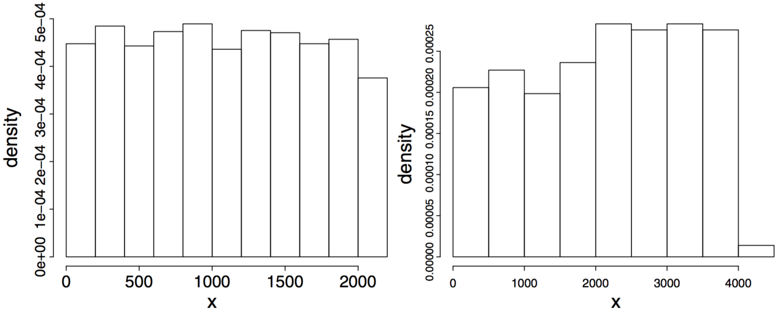
Transformed Process



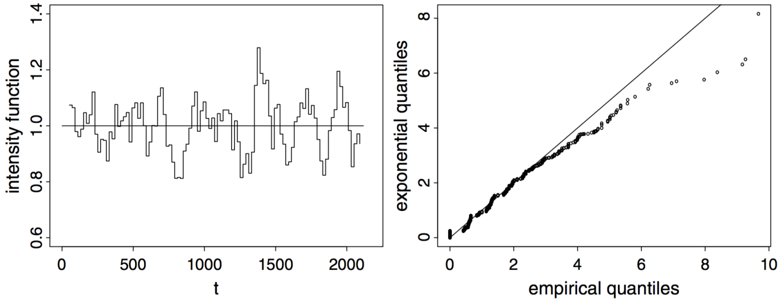
- Left: Mean value function $\mu(t)$
- Right: Transformed process $\mu(T_n)$

Histogram of values of $\mu(T_n)$ vs T_n



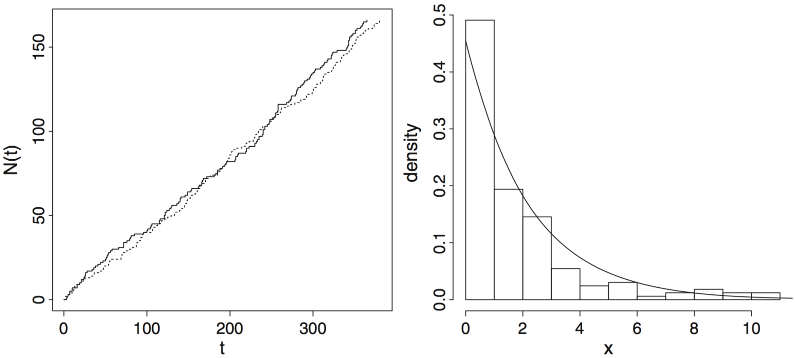
- Left: Histogram of values of $\mu(T_n)$ (looks like uniform distribution)
- Right: Histogram of values of T_n (does not look like uniform distribution)

Is Poisson Process appropriate?

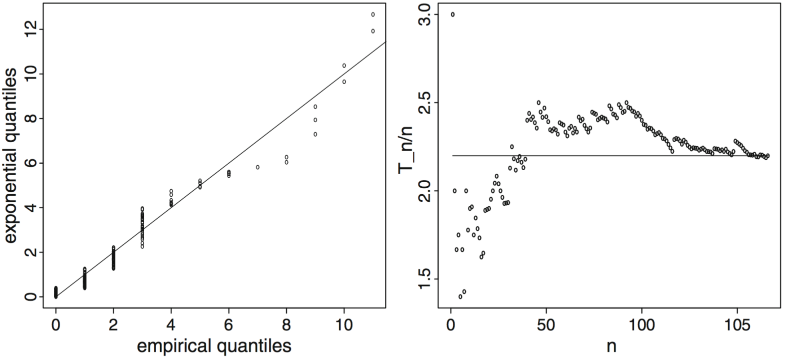


- Left: Moving average estimate of intensity function corresponding to the transformed sequence $\mu(T_n)$ (note that it fluctuates a lot)
- Right: QQ-plot of $\mu(T_n) - \mu(T_{n-1})$ against the Exp(1) (in data we see heavier tail than the one of exponential distribution)

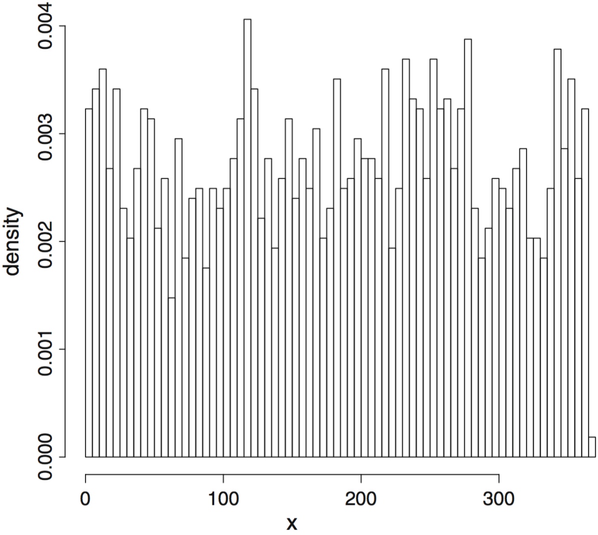
1980 Data only



- Left: Data vs One sample path of Poisson with $\hat{\lambda}^{-1} = 2.19$
- Right: Histogram of inter-arrival times vs. $Exp(\lambda)$



- Left: QQ-plot of $T_n - T_{n-1}$ against the $Exp(\lambda)$ (not great but not horrible either)
- Right: For a homogenous Poisson process: $\frac{T_n}{n} \to \lambda^{-1}$ a.s. by the SLLN



0.004
0.003
0.002
0.001
0.000
density
0
100
200
300
x