

Deň otvorených dverí

Matematika mnohorozmerných dát

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Tri príklady viacrozmerých dát

- Tváre
- Jedlo
- Sedemboj

Redukcia dimenzie je užitočná.

Rôznym spôsobom.

Tváře



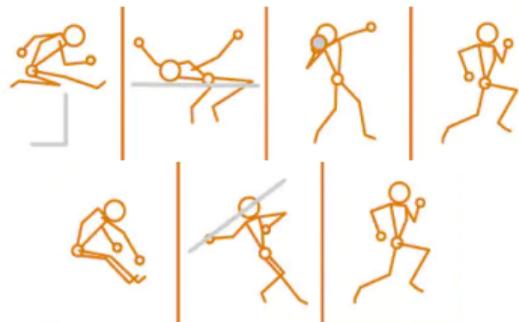
- 12000 pixelov

Jedlo



- 4 druhov živín

Sedemboj



- 7 športov

Príklad 1: tváre

Operácie s maticami

Matice môžu byť: veľké, malé, úzke, široké

Rovnako veľké matice môžeme napr. sčítať alebo odčítať.



$$\text{Smiley Face}^T = \text{Upside Down Smiley Face}$$

$$\text{Smiley Face} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \text{Upside Down Smiley Face}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \text{Smiley Face} = \text{Upside Down Smiley Face}$$

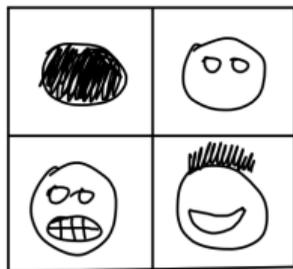
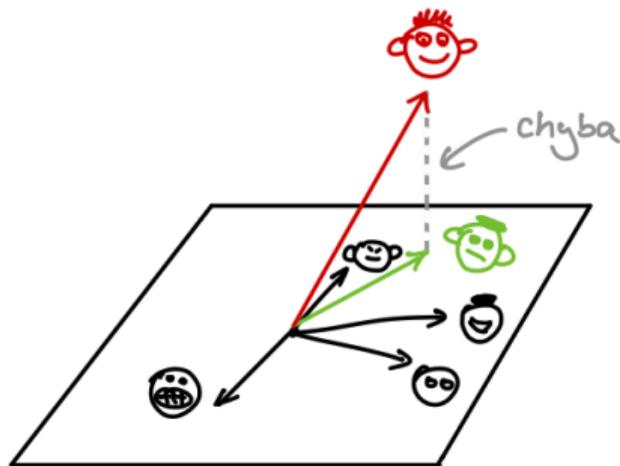
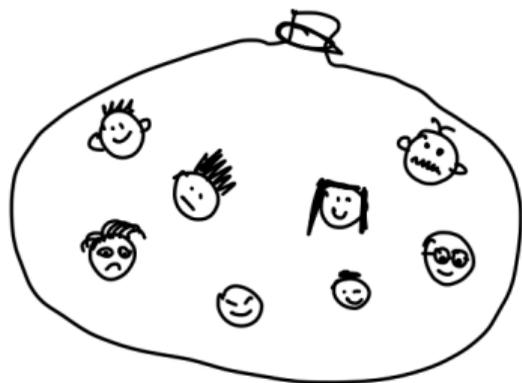
$$\left(\text{Smiley Face} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \right)^T = \text{Upside Down Smiley Face}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \text{Upside Down Smiley Face}^T = \text{Smiley Face}$$

Medzi maticami vieme merať vzdialenosti

$$\| \text{Smiley} - \text{Smiley} \| = 0,01$$

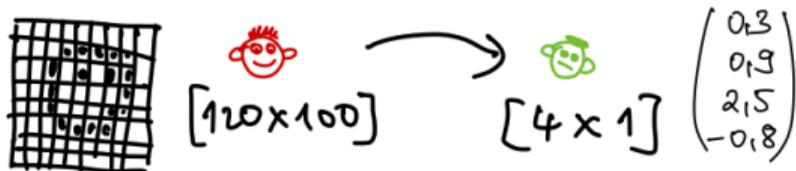
$$\| \text{Smiley} - \text{Angry Face} \| = 2,3$$



bázičné
tváre

$$\text{green face} = 0,3 \text{ eye} + 0,9 \text{ smile} + 2,5 \text{ hair} - 0,8 \text{ teeth}$$

$$\text{red face} = \text{green face} + \text{chyba}$$



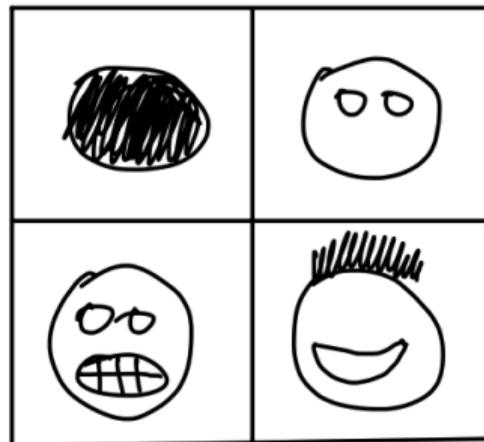
Rekonštrukcia pomocou 36 hlavných komponentov

Sample of original faces before running PCA:



Zdroj: <https://github.com/gbuesing/pca/tree/master/examples>

'Bázické tváre'



bázické
tváre

...matematika

$P = 10304$

$n = 400$

X

faces-pca\$rot

400

10304

eigenfaces

$S_{v_i} = X^T X v_i = \lambda_i v_i$ $\leftarrow p \times 1$

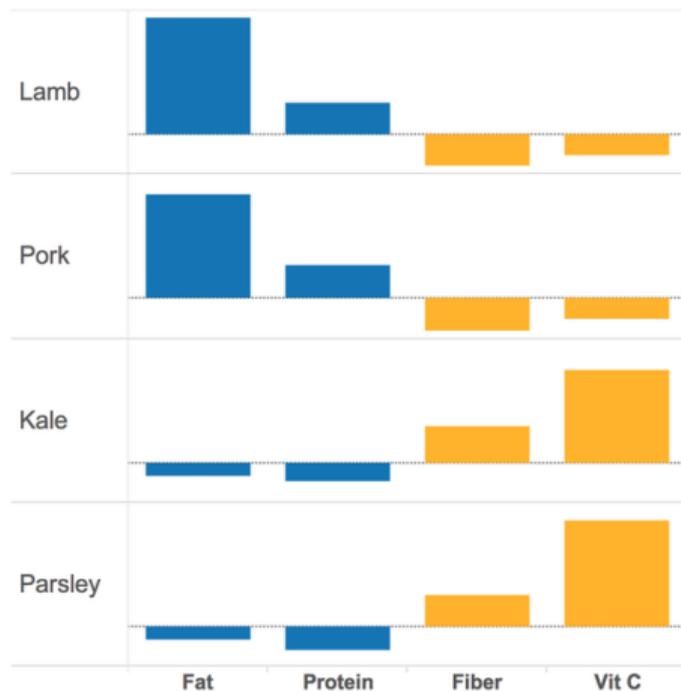
$XX^T u_i = \lambda_i u_i$ $\leftarrow n \times 1$ easy to get

$X^T X \boxed{X^T u_i} = \lambda_i \boxed{X^T u_i}$

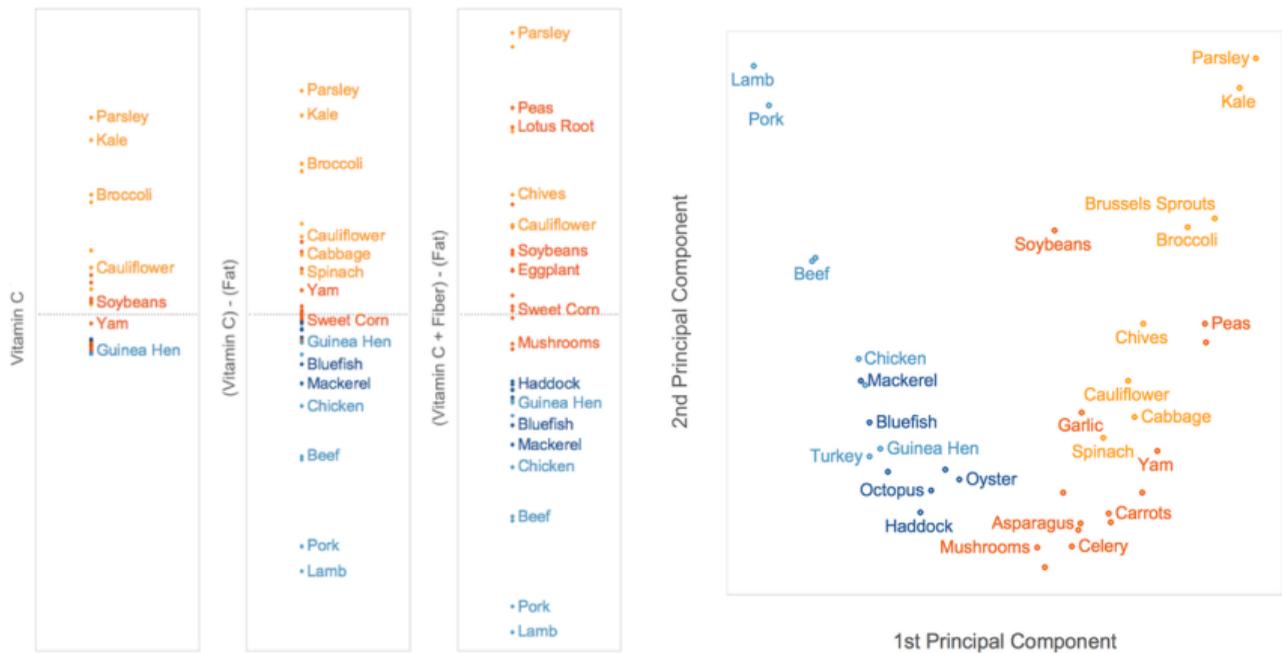
$\text{face} = \beta_0 + \beta_1 \text{eigenface}_1 + \beta_2 \text{eigenface}_2 + \beta_3 \text{eigenface}_3 + \beta_4 \text{eigenface}_4 + \epsilon$

Príklad 2: jedlo

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Obr.: Zdroj: <https://www.quora.com/What-is-an-intuitive-explanation-for-PCA>



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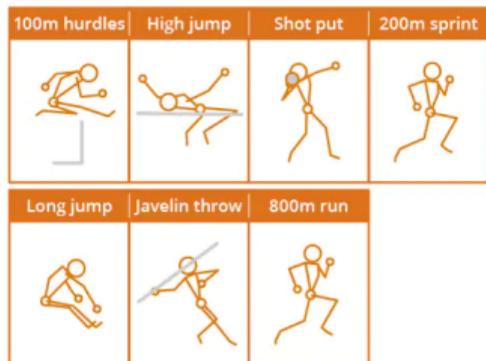
Guinea Hen



Obr.: Zdroj: wiki

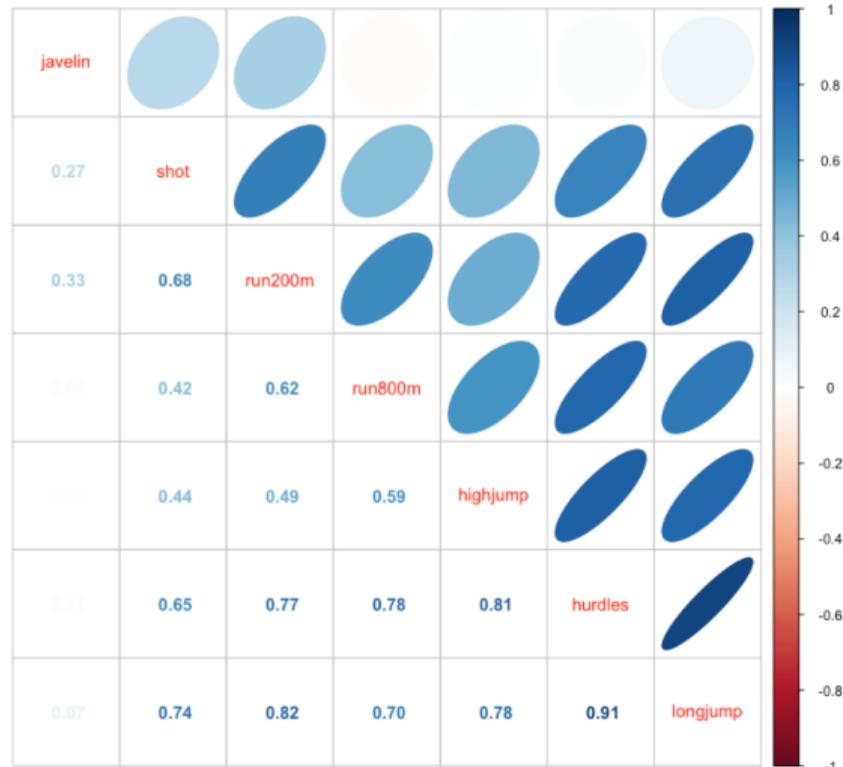
Príklad 3: sedemboj

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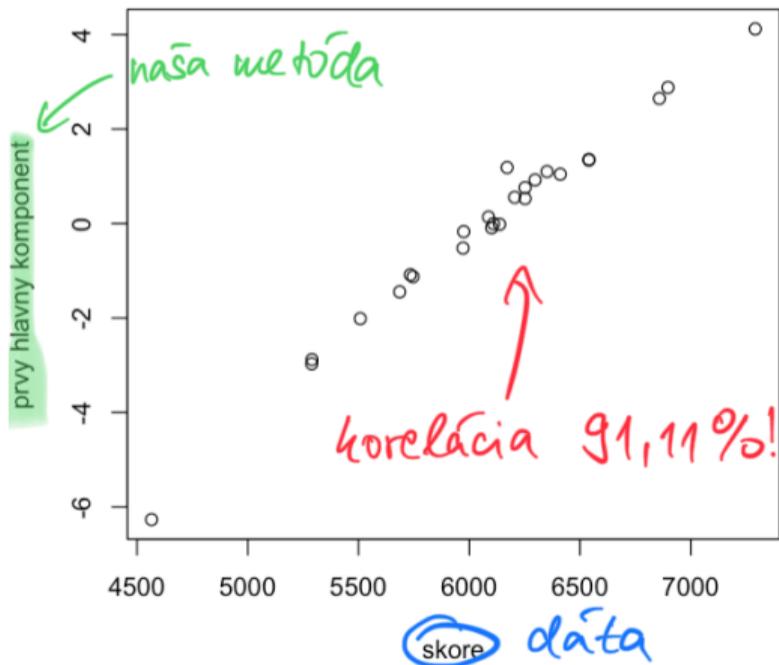
> heptathlon

| | hurdles | highjump | shot | run200m | longjump | javelin | run800m |
|---------------------|---------|----------|-------|---------|----------|---------|---------|
| Joyner-Kersey (USA) | 3.73 | 1.86 | 15.80 | 4.05 | 7.27 | 45.66 | 34.92 |
| John (GDR) | 3.57 | 1.80 | 16.23 | 2.96 | 6.71 | 42.56 | 37.31 |
| Behmer (GDR) | 3.22 | 1.83 | 14.20 | 3.51 | 6.68 | 44.54 | 39.23 |
| Sablovskaitė (URS) | 2.81 | 1.80 | 15.23 | 2.69 | 6.25 | 42.78 | 31.19 |
| Choubenkova (URS) | 2.91 | 1.74 | 14.76 | 2.68 | 6.32 | 47.46 | 35.53 |
| Schulz (GDR) | 2.67 | 1.83 | 13.50 | 1.96 | 6.33 | 42.82 | 37.64 |
| Fleming (AUS) | 3.04 | 1.80 | 12.88 | 3.02 | 6.37 | 40.28 | 30.89 |
| Greiner (USA) | 2.87 | 1.80 | 14.13 | 2.13 | 6.47 | 38.00 | 29.78 |
| Lajbnerova (CZE) | 2.79 | 1.83 | 14.28 | 1.75 | 6.11 | 42.20 | 27.38 |
| Bouraga (URS) | 3.17 | 1.77 | 12.62 | 3.02 | 6.28 | 39.06 | 28.69 |
| Wijnsma (HOL) | 2.67 | 1.86 | 13.01 | 1.58 | 6.34 | 37.86 | 31.94 |
| Dimitrova (BUL) | 3.18 | 1.80 | 12.88 | 3.02 | 6.37 | 40.28 | 30.89 |
| Scheider (SWI) | 2.57 | 1.86 | 11.58 | 1.74 | 6.05 | 47.50 | 28.50 |
| Braun (FRG) | 2.71 | 1.83 | 13.16 | 1.83 | 6.12 | 44.58 | 20.61 |
| Ruotsalainen (FIN) | 2.63 | 1.80 | 12.32 | 2.00 | 6.08 | 45.44 | 26.37 |
| Yuping (CHN) | 2.49 | 1.86 | 14.21 | 1.61 | 6.40 | 38.60 | 16.76 |
| Hagger (GB) | 2.95 | 1.80 | 12.75 | 1.14 | 6.34 | 35.76 | 24.95 |
| Rynn (USA) | 2.35 | 1.83 | 12.69 | 1.78 | 6.13 | 44.34 | 17.00 |



Chceli by sme tento 7 rozmerný priestor zredukovať do 1 rozmerného.

Ako veľmi bude toto číslo podobné skutočnému skóre?



- Naša metóda zredukovala 7D do 1D
- Nikdy nevidela **dáta** o nameranom skóre (!!!)
- Napriek tomu sme dostali **skoro totožné výsledky**
- Interpretácia?

Zhrnutie

Tváre



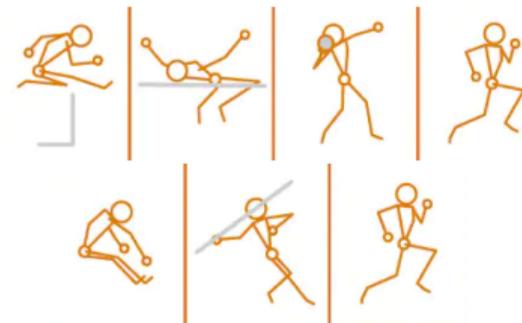
- 12000D → 36D
- úspora pamäte

Jedlo



- 4D → 2D
- vizualizácia

Sedemboj



- 7D → 1D
- porozumenie

Ďakujem za pozornosť!



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